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# FINITE-ELEMENT MODELS FOR CALCULATING THE TEMPERATURE FIELDS OF UNDERGROUND PIPELINES

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A simple method of constructing finite elements for the numerical modeling of temperature fields of underground pipelines is outlined.

In the practical design of mains pipelines, problems of temperature-field calculation are of particular interest [1]. Variations in the temperature conditions of the pipeline and the surrounding material exert an influence on the stress—strain state of the tube and cause settling of the earth, which leads to stability loss of the pipeline and other undesirable consequences [2]. Recently, there has been a trend to more completely taking account of the whole set of real conditions of pipeline use. In connection with this, there has been a significant increase in the role of numerical methods of calculation with the use of a computer. The most flexible and universal method is the finite-element method (FEM), which presently occupies the central position in engineering calculations [3]. FEM allows the thermal interaction of the pipeline with surrounding material of inhomogeneous structure to be analyzed [4], and allows the influence of imperfections and damage in the insulation and other anomalies in the thermophysical characteristics to be taken into account.

The present work outlines a simplified method of constructing finite elements (FE), taking account of the geometry of the problem and allowing errors of boundary approximation to be eliminated. In this case, plane FE in polar coordinates (Fig. 1) and three-dimensional FE in cylindrical coordinates (Fig. 2) are most appropriate.

Constructing an interpolational polynomial for the FE entails selecting an appropriate system of finite basis functions. The temperature values $T_i$ at corners of the FE are taken as the interpolation parameters. The problem reduces to constructing the coordinate functions $\Phi_i$. Usually [3], this problem involves investigating a system of linear algebraic equations, the order of which is equal to the number of degrees of freedom of the FE. Solving such systems by means of matrix algebra entails laborious transformations and well-known calculational difficulties.

A simple and expedient method of geometric formalization of the FE basis is now written [5], generalizing the probability concept of baricentric coordinates of simplex models. Probability identification of the basis significantly simplifies the procedure for constructing

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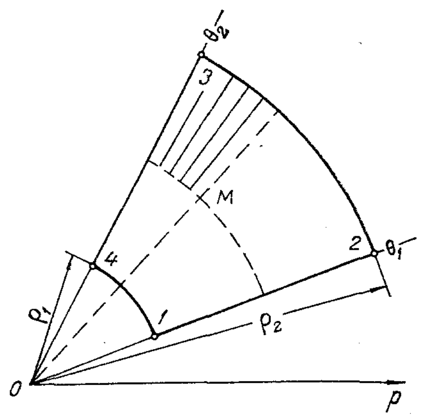


Fig. 1

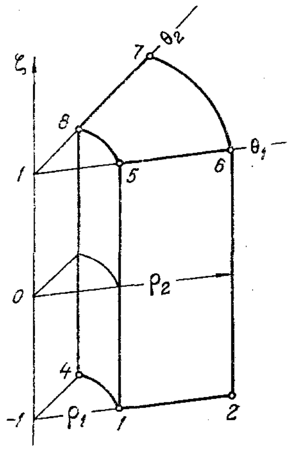


Fig. 2

Fig. 1. Curvilinear FE in polar coordinates.

Fig. 2. Three-dimensional FE in cylindrical coordinates.

various FE models in one-, two-, and three-dimensional problems. As an example, the construction of a basis function corresponding to point 1 of a plane FE is demonstrated (Fig. 1). The current point $M(\rho, \theta)$ is chosen on the element. Point 1 is positioned in accordance with the opposite region bounded by coordinate lines including the point M (this region is shaded in Fig. 1). To write $\Phi_1$, it is now sufficient to find the probability that a random point will fall in the shaded region. An analogous method of calculating geometric probabilities is used for three-dimensional models (Fig. 2). If the third coordinate is normalized $(-1\leqslant\zeta\leqslant 1)$, the basis functions of the three-dimensional FE take the form

$$\Phi_1 = \frac{\rho/\rho_1-\rho_0}{1-\rho_0}\,\frac{\theta_2-\theta}{\theta_0}\,\frac{1-\zeta}{2},$$
$$\Phi_2 = \frac{\rho/\rho_1-1}{\rho_0-1}\,\frac{\theta_2-\theta}{\theta_0}\,\frac{1-\zeta}{2},$$
$$\Phi_3 = \frac{\rho/\rho_1-1}{\rho_0-1}\,\frac{\theta-\theta_1}{\theta_0}\,\frac{1-\zeta}{2},$$
$$\Phi_4 = \frac{\rho/\rho_1-\rho_0}{1-\rho_0}\,\frac{\theta-\theta_1}{\theta_0}\,\frac{1-\zeta}{2}, \quad (1)$$

where $\rho_0 = \rho_2/\rho_1$; $\theta_0 = \theta_2 - \theta_1$.

To obtain $\Phi_i$ at the points 5, 6, 7, 8 of the upper face of the FE, it is sufficient to reverse the sign of $\zeta$ in Eq. (1). The interpolational formula for the temperature at the element is written in the form

$$T(\rho,\ \theta,\ \zeta) = \sum_{i=1}^{8}\Phi_i T_i, \quad \sum_{i=1}^{8}\Phi_i = 1.$$

When $\zeta = -1$, the basis of the plane FE in polar coordinates is obtained from Eq. (1) (Fig. 1).

The interpolational quality of the models constructed is well illustrated by a simple test of the steady point-by-point distribution of the heat of the internal source generated in the element. For example, calculations for a three-dimensional element with the geometric characteristics $1\leqslant\rho/\rho_1\leqslant 2$, $0\leqslant\theta\leqslant\pi/2$, $-1\leqslant\zeta\leqslant 1$ show that the heat distributed over the points is nonuniform, as would be expected. Points 1, 4, 5, 8 on the internal cylindrical surface of the FE each reproduce 11/72 of the heat, while points 2, 3, 6, 7 account for 17/72 of the heat.

Note, in conclusion, that the geometric method of constructing the FE basis, distinguished by simplicity and universality, is especially effective in modeling elements of higher orders of approximation.

## NOTATION

T, temperature; $T_i$, nodal temperature values; $\Phi_i$, basis functions of the finite element; ρ, θ, ζ, cylindrical coordinates.

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# SOLVING NONSTEADY HEAT-CONDUCTION PROBLEMS FOR MULTILAYER SYSTEMS BY THE FINITE-DIFFERENCE METHOD

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The problem of heat propagation in a multilayer system with intrinsic heat liberation of any subsystem depending on the temperature, coordinates, and time is considered.

The construction of concrete housings of underground structures in various climatic conditions requires optimization of the temperature conditions of concreting. In practice, temperature regulation is accomplished either by producing definite conditions of concrete heating in the jacket (thermal-heating method) or by heating the housing by means of insulational materials in contact with air (thermos method). Theoretical analysis of the choice of parameters of the optimal conditions reduces to solving the problem of nonsteady heat conduction in a multilayer system.

Suppose that, within the limits of each subsystem, the thermophysical characteristics are constants and the heat liberation of any subsystem may be represented by a specified function of the temperature, time, and coordinates Q = Q(U, r, t). Then heat propagation in the system may be described by the following nonlinear equation of nonsteady heat conduction for the one-dimensional case in cylindrical coordinates

$$\frac{r}{c_v}\frac{\partial Q}{\partial t} = r\frac{\partial U}{\partial t} - \frac{\partial}{\partial r}\left(ar\frac{\partial U}{\partial r}\right), \quad r_{\mathrm{L}} < r < r_{\mathrm{re}} \tag{1}$$

with initial condition

$$U(r, 0) = \varphi(r), \quad r_{\mathrm{L}} < r < r_{\mathrm{re}} \tag{2}$$

and boundary conditions of the first kind

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